UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2013/2014 Academic Session

June 2014

# ESA 322/3 – Structural Dynamics
## *[Dinamik Struktur]*

Duration : 3 hours
*[Masa : 3 jam]*

Please check that this paper contains **TWENTY (20)** printed pages, **THREE (3)** pages appendix and **SIX (6)** questions before you begin the examination.

*[Sila pastikan bahawa kertas soalan ini mengandungi **DUA PULUH (20)** mukasurat bercetak, **TIGA (3)** mukasurat lampiran dan **ENAM (6)** soalan sebelum anda memulakan peperiksaan.]*

**Instructions** : Answer **SIX (6)** questions.

*[**Arahan** : Jawab **ENAM (6)** soalan].*

Student may answer the questions either in English or Bahasa Malaysia.

*Pelajar boleh menjawab soalan dalam Bahasa Inggeris atau Bahasa Malaysia.*

Answer to each question must begin from a new page.

*[Jawapan untuk setiap soalan mestilah dimulakan pada mukasurat yang baru.]*

**Appendix**/*Lampiran :*

1 Fundamental Equations in Vibration [1 page/*mukasurat*]
2. Laplace Transform Pairs [1 page/*mukasurat*]
3. Vibration-Related Formulas [1 page/*mukasurat*]

In the event of any discrepancies, the English version shall be used.

*[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]*

Answer **SIX (6)** questions.
*Jawab **ENAM (6)** soalan.*

1. [a] Answers the following questions in words. For questions (i) and (ii), complement your answers with the relevant equations if required.

   (i) Define the meaning of simple harmonic motion?

   (ii) State the difference between undamped and damped natural frequencies

   (iii) Define the meaning of mode shape?

**(30 marks)**

[b] Plot vibration response that illustrates a single degree of freedom system undergoing free oscillation for each case;

   (i) No damping (undamped)

   (ii) Under damped, $\zeta<1$

   (iii) Critically damped, $\zeta=1$

   (iv) Over damped, $\zeta>1$

Responses must be plotted on the same axis and label clearly.

**(20 marks)**

[c]

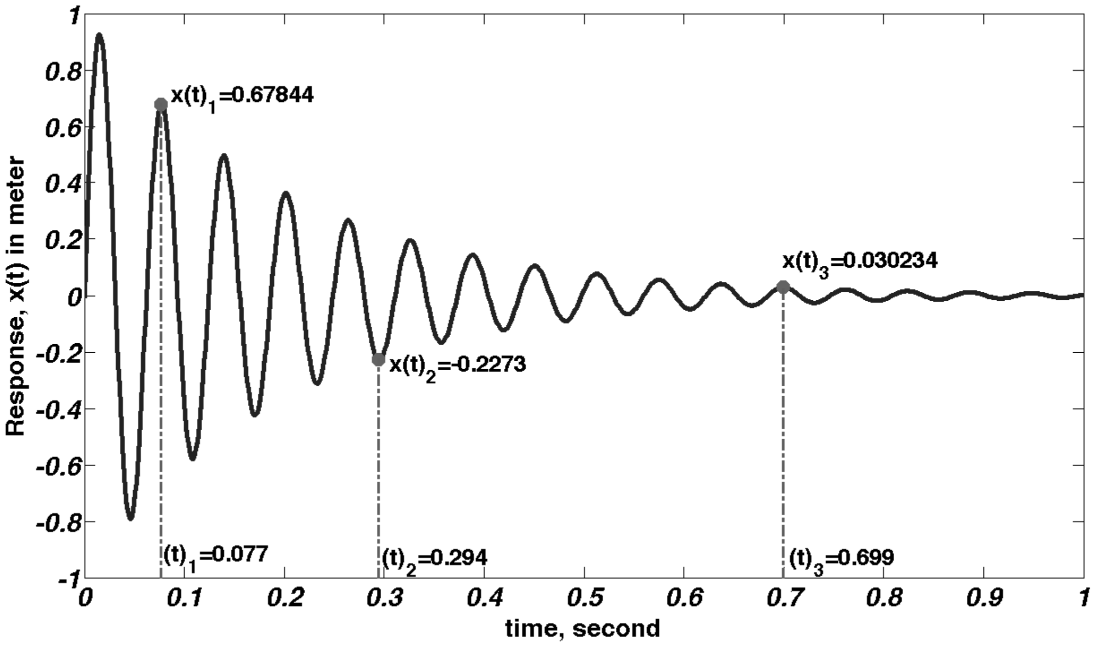


**Figure 1**

Figure 1 shows free response for a single degree of freedom system undergoing decay oscillation due to the existence of viscous damping. Based on figure 1 and by using appropriate equations, compute:

(i) Period of oscillation of the system in second.

(ii) Damped natural frequency of the system in rad/s.

(iii) Damping ratio of the system.

(iv) Undamped natural frequency of the system in rad/s.

(v) Viscous damping coefficient of the system in N-s/m if mass m =2kg.

**(50 marks)**

2. [a]

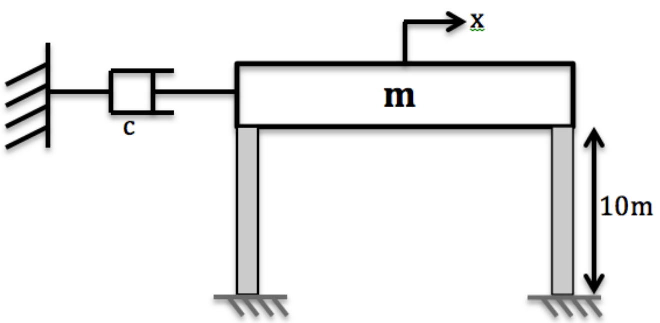



**Figure 2**

Figure 2 shows a building consisting of a rigid floor of mass *m* and two massless columns. At their upper end, the columns are fully clamped with the floor that is connected to a damper. At their lower end they are fix on the ground. The floor can only oscillates horizontally due to the flexibility of the columns. You must:

(i) Sketch a free body diagram of the system.

(ii) Derive the equation of motion of the system in terms of m, c and EI expression where the stiffness of each column is given as;

$$k = \frac{12EI}{h^3}$$

(iii) Find the natural frequency of the floor in Hz given EI = 20,000Nm$^2$ and mass m = 100kg.

(iv) Approximate the floor steady state amplitude if the system is excited at 0.02Hz with harmonic force $F_o$= 450N acting on the mass. Assume the value of viscous damping coefficient to be 100N-s/m.

(v) If the harmonic excitation is turned off, will the system's oscillation frequency remains the same, increase or decrease. Give reasons to your answer.

**(50 marks)**

[b] A small wind turbine assembly is to be installed on a building roof to generate extra electrical energy. The entire assembly has a mass of 290kg. One of the turbine blades has an unbalance mass of 2.5kg with 0.1m of eccentricity. The wind turbine will rotate at constant speed of 875 rpm. The assembly will be placed on four springs arranged in parallel with each has 261,000N/m of stiffness.

(i) Predict the undamped natural frequency of the entire assembly.

(ii) Compute the force amplitude acting on the system resulted from the unbalanced mass when the wind turbine is rotating.

(iii) Predict the amplitude of oscillation of the assembly if there is no damping exists in the system.

(iv) If there is a damper with damping ratio value of $\zeta$ = 0.001 and the blades rotate at the same frequency as its natural frequency, estimate the amplitude of the assembly?

(v) If the wind turbine blades suddenly stop rotating. Will the wind turbine assembly continue to oscillate or not? Provide reason/s for your answer.

**(50 marks)**

3. [a] A golf cart is moving through the golf course at 10km/hr. The track surface of the golf's course can be represented as sinusoid as shown in figure 3. Given the mass of the golf cart is equal to 100kg, determine:

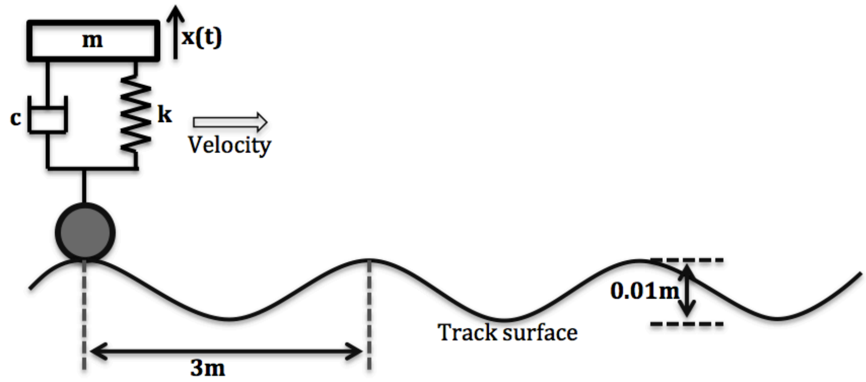



**Figure 3**

(i) Excitation function y(t) generated from the terrain.

(ii) The cart's oscillation frequency given k = 4000N/m and c = 200Ns/m.

(iii) The cart's response amplitude when moving on this terrain at 10km/hr.

(iv) The cart's response amplitude if the cart doubles its speed.

(v) The cart's response amplitude if a golfer weight 75kg drives the cart at 15km/hr.

**(50 marks)**

[b] Consider a two-degree of freedom system shown in figure 4, given mass, m = 10kg and the spring stiffness, k = 40N/m, you must:

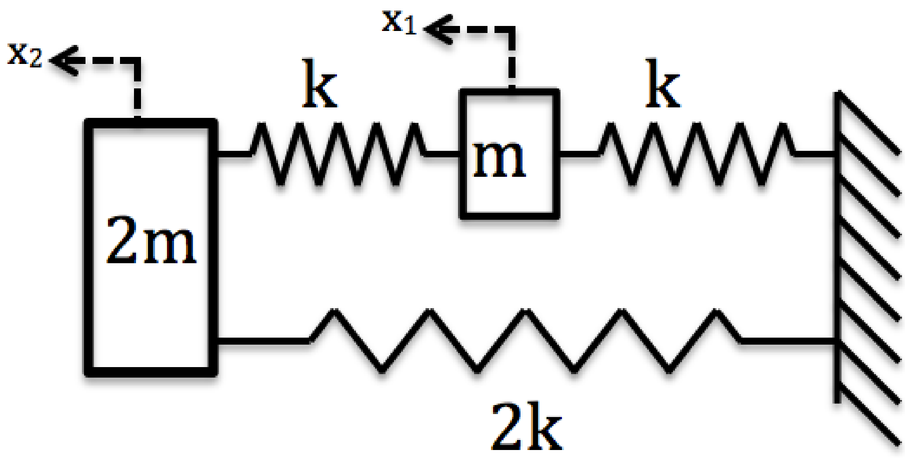



**Figure 4**

(i) Draw the free body diagram of the system.

(ii) Derive the equation of motion of the system in matrix form.

(iii) Determine the characteristic equation.

(iv) Calculate the natural frequencies.

(v) Calculate and draw the mode shapes.

**(50 marks)**

4. [a] With the help of Collar's triangle, write short notes on the following aeroelastic phenomena.

(i) Divergence

(ii) Control reversal

(iii) Stall flutter

**(30 marks)**

[b] A typical airfoil section that illustrates a divergence model of a simple wing is shown in figure 5. Proof that the expression for divergence dynamic pressure $q_D$ is

$$q_D = \frac{K}{S\left(\frac{\delta C_L}{\delta \alpha}\right) e}$$

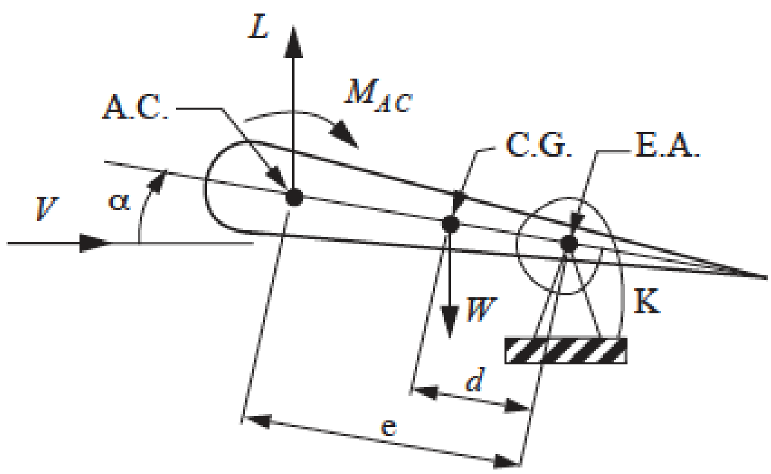



**Figure 5**

**(40 marks)**

5. [a] An airfoil section with mass $M_m$ and Inertia $I_I$ shown in Figure 6 undergoes a combined bending and torsion modes of oscillation. The stiffness for the two-degree of freedom system is provided by linear and torsional springs. Obtain the equation of motion of the system using Lagrange's equation assuming there are aerodynamic forces (Lift L and pitching moment M) acting on the airfoil.

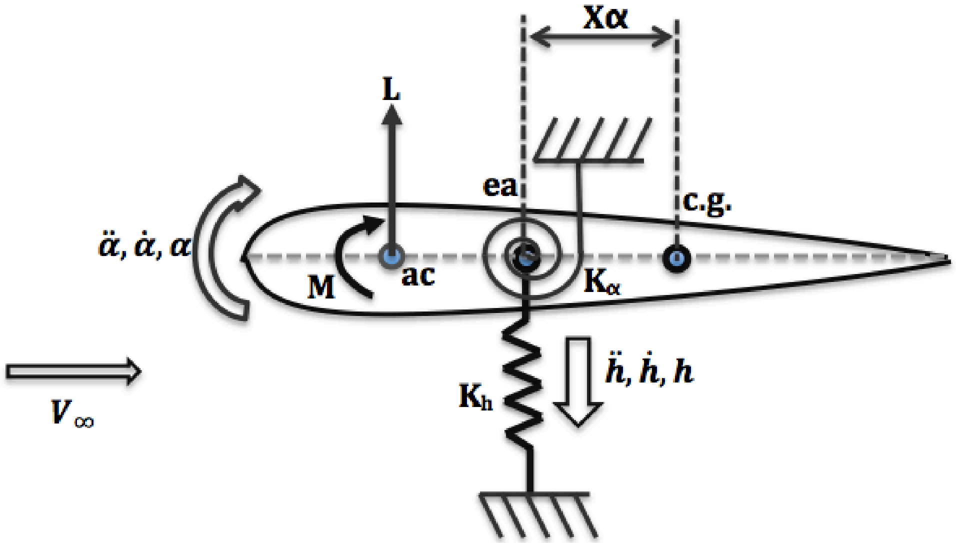



**Figure 6**

**(60 marks)**

[b] List three passive flutter suppression techniques that could be incorporated in an aircraft in order to improved flutter safety margin. Give details.

**( 30 marks)**

[c] (i) Describe general procedure for performing flight flutter testing.

(ii) Plot typical curves illustrating damping against airspeed (v-g) and frequency against airspeed (v-f) obtained from analyzing result of flutter experiment in the wind tunnel. Indicate the flutter point in the plot and label your plot clearly.

**(40 marks)**

6. The Laplace Transform can be used to determine the transfer function of a dynamic system and it is one of the methods apart from the solution in time domain to determine the response of the system for any given input.

[a] In your opinion in solving vibration problems, which method is preferable and why?

**(10 marks)**

[b] Formulate the transfer function of a single degree of freedom system with mass m, stiffness k and damping c when subjected to a general forcing function, f(t)

**(40 marks)**

[c] For the value of m=1 kg, c=10 Ns/m and k=21 N/m, determine the poles of the system. What is the significance of these values?

**(30 marks)**

[d] Draw the block diagram of this system.

**(20 marks)**

1. [a] *Berikan jawapan kepada soalan berikut dalam bentuk perkataan. Untuk soalan (i) dan (ii), sertakan persamaan yang berkenaan jika perlu.*

   (i) *Apakah definisi gerakan mudah harmonik?*

   (ii) *Nyatakan perbezaan diantara frekuensi tabii tidak teredam dan frekuensi tabii teredam?*

   (iii) *Apakah yang dimaksudkan dengan bentuk mod?*

**(30 markah)**

   [b] *Plot kelakuan getaran yang mengambarkan sistem satu darjah kebebasan yang menjalani getaran bebas untuk setiap kes berikut;*

   (i) *Tiada redaman (tidak teredam)*

   (ii) *Kurang redam,* $\zeta<1$

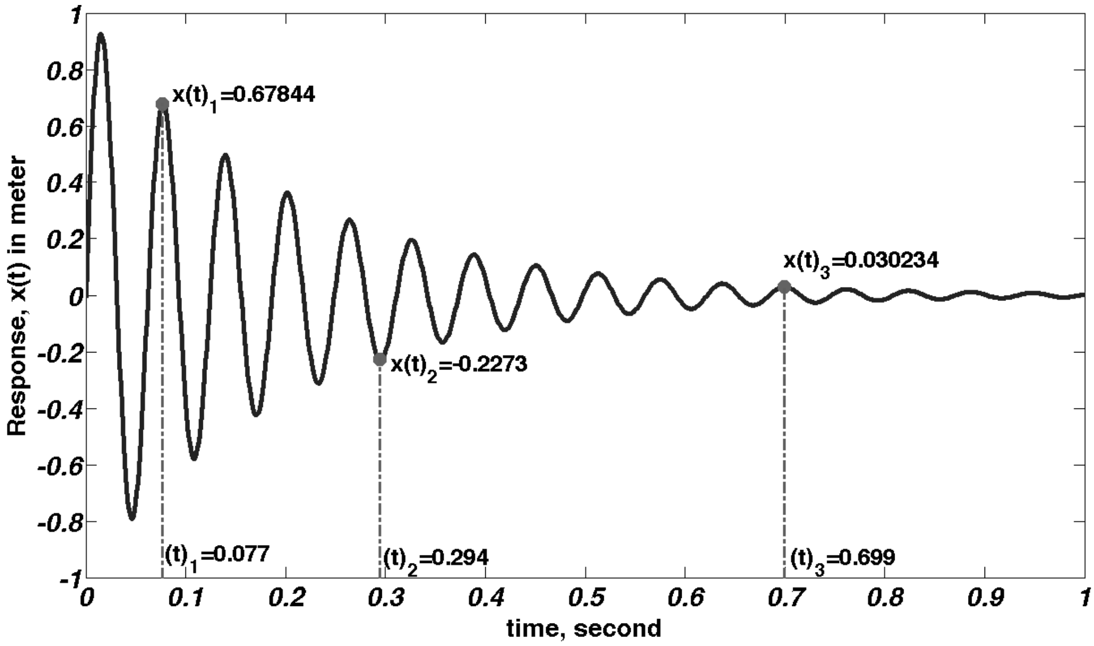
   (iii) *Redaman kritikal,* $\zeta=1$

   (iv) *Lebih redam,* $\zeta>1$

   *Sambutan getaran mesti di plot di dalam satu paksi serta dilabel dengan jelas*

**(20 markah)**

*[c]*




**Rajah 1**

*Rajah 1 menunjukkan getaran bebas untuk sistem satu darjah kebebasan yang mengalami getaran redaman disebabkan wujudnya redaman likat. Berdasarkan Rajah 1 dan dengan menggunakan persamaan persamaan yang berkenaan, kirakan:*

*(i)* *Tempoh getaran sistem tersebut dalam unit saat.*

*(ii)* *Frekuensi tabii teredam sistem tersebut dalam unit rad/.*

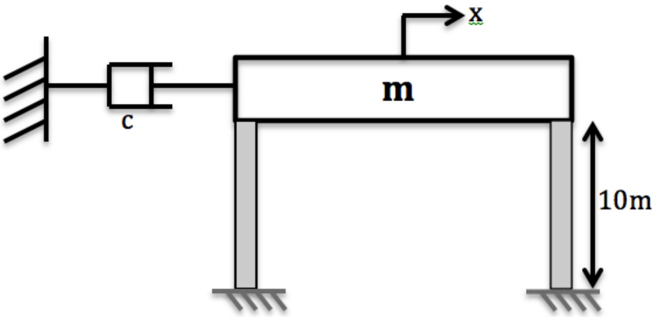
*(iii)* *Nisbah redaman sistem tersebut.*

*(iv)* *Frekuensi tidak teredam sistem tersebut dalam unit rad/s.*

*(v)* *Pekali redaman likat sistem tersebut dalam unit N-s/m jika jisim m = 2kg.*

***(50 markah)***

2. *[a]*




**Rajah 2**

*Rajah 2 menunjukkan sebuah bangunan yang mengandungi lantai tegar berjism* ***m*** *dan dua tiang tanpa jisim. Lantai diapit pada bahagian atas tiang tiang tersebut serta disambung pada peredam. Tiang tiang tersebut di matikan pada bahagian tapak. Lantai tersebut hanya boleh menjalani getaran mendatar disebabkan keanjalan tiang tiang tersebut. Anda mesti:*

(i) *Lakarkan rajah badan bebas sistem tersebut.*

(ii) *Terbitkan persamaan gerakan sistem tersebut dengan menggunakan ungkapan m, c and EI sahaja, di mana keanjalan setiap tiang diberi sebagai;*

$$k = \frac{12EI}{h^3}$$

(iii) *Kirakan frekuensi tabii lantai di dalam unit Hz jika nilai $EI$=20,000Nm$^2$ dan jisim m = 100kg.*

(iv) *Kirakan amplitude lantai jika sistem itu diuja pada 0.02Hz dengan daya harmonik $F_o$= 450N dikenakan pada jisim tersebut. Andaikan nilai pekali redaman likat sebanyak 100N-s/m.*

(v) *Jika pengujaan harmonik dihentikan, adakah nilai frekuensi getaran bebas sistem tersebut akan sama, meningkat atau menurun. Berikan sebab kepada jawapan anda.*

***(50 markah)***

[b] *Sebuah turbin angin kecil akan dipasang diatas bumbung sebuah bangunan untuk menjana tenaga elektrik tambahan. Keseluruhan pemasangan mempunyai nilai jisim 290kg. Satu daripada bilah turbin mempunyai jisim tidak seimbang sebanyak 2.5kg dengan jarak 0.1 kesipian. Turbin angin akan berputar pada kelajuan 875 psm. Keseluruhan pemasangan turbin akan diletakan diatas empat spring yang disusun secara selari dengan setiap satu spring mempunyai nilai keanjalan 261,000N/m.*

*(i)* *Kirakan nilai frekuensi tabii tidak teredam keseluruhan pemasangan tersebut.*

*(ii)* *Kirakan amplitud daya yang dikenakan keatas sistem tersebut oleh kedudukan jisim yang tidak seimbang apabila turbin angin berputar.*

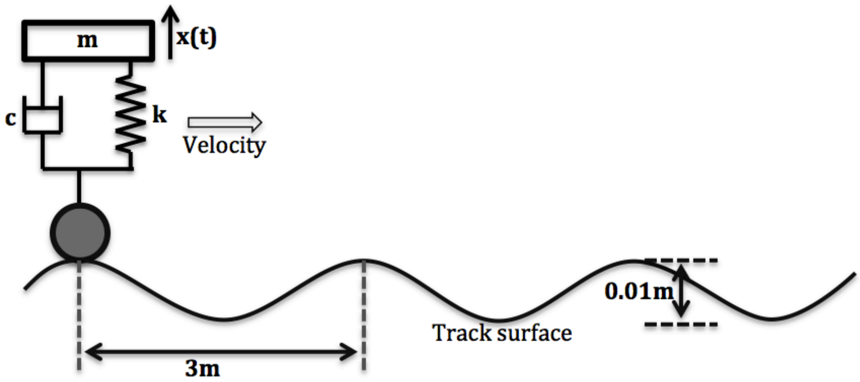
*(iii)* *Ramalkan amplitud getaran keseluruhan pemasangan jika tiada redaman bertindak keatas sistem tersebut.*

*(iv)* *Jika terdapat peredam dengan nilai nisbah redaman $\zeta = 0.001$ dan bilah turbin berputar pada frekuensi yang sama seperti frekuensi tabii, anggarkan amplitud keseluruhan pemasangan tersebut.*

*(v)* *Jika turbin angin berhenti berputar dengan serta merta, Adakah keseluruhan pemasangan turbin angin akan terus bergetar atau tidak. Sertakan alasan bersama jawapan yang anda berikan.*

***(50 markah)***

3. [a] *Sebuah kereta golf sedang bergerak melalui padang golf pada kelajuan 10km/j. Permukaan laluan kereta golf tersebut boleh diandaikan seperti lengkungan sinus seperti yang digambarkan dalam Rajah 3. Nilai jisim kereta golf yang diberikan ialah 100kg. Jika suspensi kereta golf tersebut di modelkan sebagai sistem satu darjah kebebasan, tentukan:*




**Rajah 3**

*(i) Fungsi uja y(t) yang dijana oleh permukaan laluan.*

*(ii) Frekuensi getaran apabila diberi k = 4000N/m dan c = 200Ns/m.*

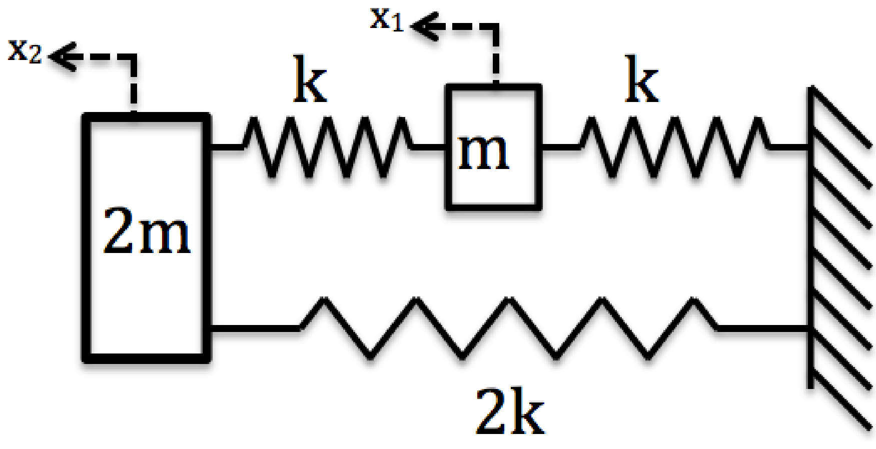
*(iii) Amplitud respon kereta golf apabila melalui permukaan padang golf tersebut dengan kelajuan 10km/sjm.*

*(iv) Amplitud respon kereta golf apabila melalui permukaan padang golf tersebut pada 2 kali ganda kelajuan yang diberikan.*

*(vi) Amplitud respon kereta golf jika pemain golf seberat 75kg memandu kereta golf tersebut pada kelajuan 15km/.j*

***(50 markah)***

[b] *Sistem dua darjah kebebasan ditunjukkan dalam Rajah 4, jika jisim m = 10kg dan keanjalan spring, k = 40N/m, anda mesti:*




**Rajah 4**

(i) *Lakarkan rajah badan bebas kedua dua jisim.*

(ii) *Terbitkan persamaan gerakan sistem ddalam bentuk matriks.*

(iii) *Tentukan persamaan ciri.*

(iv) *Kirakan frekuensi frekuensi tabii.*

(v) *Kirakan bentuk bentuk mod dan lakarkan.*

***(50 markah)***

4. *[a]* *Dengan berpandukan segitiga Collar , tulis nota ringkas mengenai fenomena-fenomena aeroleastik berikut;*

*(i)* *Kecapahan*

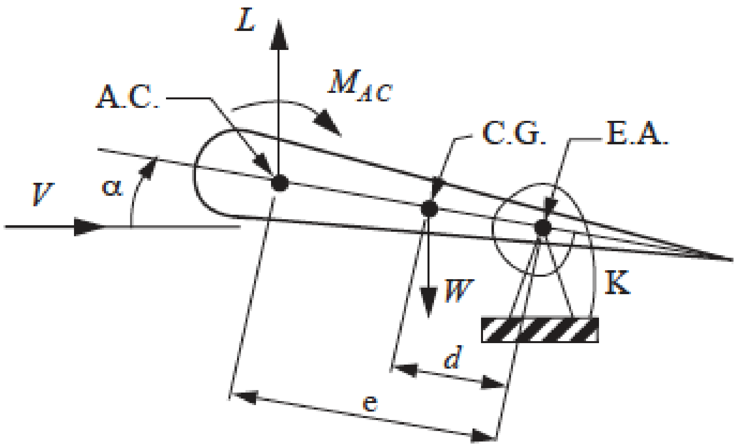
*(ii)* *Kawalan pembalikan*

*(iii)* *Kibaran stall*

***( 30 markah)***

*[b]* *Satu model kerajang sayap mewakili fenomena kecapahan sayap mudah ditunjukkan dalam Rajah 5. Buktikan ungkapan tekanan dinamik kecapahan* $q_D$ *adalah seperti*

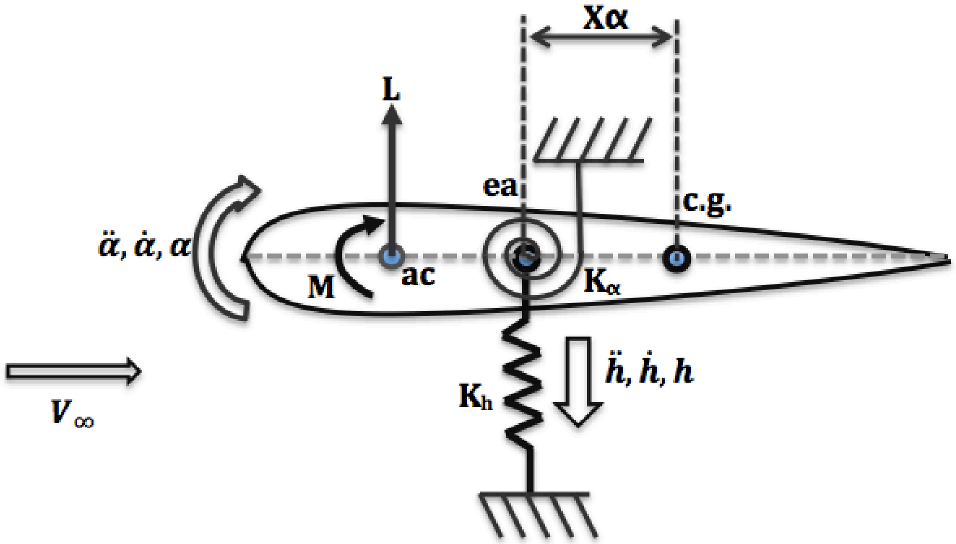
$$q_D = \frac{K}{S\left(\frac{\delta C_L}{\delta \alpha}\right)e}$$




**Rajah 5**

***( 40 markah)***

5. [a] *Satu kerajang sayap dengan jisim $M_m$ dan Inertia $I_I$ ditunjukkan dalam Rajah 6 mengalami gerakan lenturan dan kilasan. Keanjalan untuk sistem dua darjah kebebasan tersebut dibekalkan oleh spring linear dan kilasan.*
*Terbitkan persamaan gerakan sistem tersebut dengan menggunakan kaedah persamaan"Lagrange"dengan mengandaikan terdapat daya-daya aerodinamik (daya angkat L dan momen angguk M) bertindak keatas kerajang sayap tersebut.*




***Rajah 6***

***(60 markah)***

[b] *Senaraikan tiga teknik penindasan kibaran yang boleh digunapakai di dalam pesawat terbang untuk memperbaiki margin keselamatan kibaran. Berikan maklumat lengkap.*

***(30 markah)***

[c] [i] *Perihalkan prosedur umum untuk melakukan ujian kibaran terbang.*

[ii] *Plotkan lengkungan tipikal redaman dan frekuensi lawan halaju yang diperolehi daripada analisa keputusan eksperimen kibaran di dalam terowong angin. Tunjukkan titik kibaran di dalam plot dan labelkan plot dengan jelas*

***(40 markah)***

6. *Kaedah jelmaan Laplace boleh digunakan untuk menentukan rangkap pindah bagi sebuah sistem dinamik dan ianya merupakan satu kaedah selain daripada penyelesaian di dalam domain masa untuk menentukan tindakbalas sistem apabila dikenakan sebarang bentuk daya.*

[a] *Pada pendapat anda di dalam menyelesaikan masalah getaran, kaedah manakah yang lebih mudah digunakan dan kenapa?*

***(10 markah)***

[b] *Terbitkan rangkap pindah bagi sebuah system satu darjah kebebasan dengan jisim m, kekakukan k dan redaman c apabila dikenakan satu fungsi daya umum f(t)*

***(40 markah)***

[c] *Bagi nilai-nilai m=1 kg, c=10 Ns/m and k=21 N/m, tentukan nilai kutub-kutub sistem. Apakah kepentingan nilai ini?*

***(30 markah)***

[d] *Lukis rajah blok bagi sistem ini.*

***(20 markah)***

**APPENDIX 1 /** *LAMPIRAN 1*

## Fundamental Equations in Vibration

1. $\zeta = \dfrac{c}{2m\omega_n}$;

2. $x(t) = Ce^{-\zeta\omega_n t}\sin(\omega_d t + \psi)$; $\omega_d = \sqrt{1-\zeta^2}\,\omega_n$

$$C = \sqrt{x_0^2 + \frac{(\dot{x}_0 + \zeta\omega_n x_0)^2}{(1-\zeta^2)\omega_n^2}} \quad ; \quad \psi = \tan^{-1}\frac{\sqrt{1-\zeta^2}\,\omega_n x_0}{\dot{x}_0 + \zeta\omega_n x_0}$$

3. For $F(t) = me\omega^2 \sin\omega t$

$$X = \frac{me\omega^2}{\sqrt{(k-M\omega^2)^2 + (c\omega^2)^2}}; \quad \phi = \tan^{-1}\left[\frac{c\omega}{k-M\omega^2}\right]$$

$$\frac{F_T}{F_0} = \left[\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}\right]^{1/2}$$

5. For base excitation

$$\frac{X}{Y} = \left[\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}\right]^{1/2} \qquad \phi = \tan^{-1}\left[\frac{2\zeta r^3}{1+(4\zeta^2-1)r^2}\right]$$

$$\frac{F_T}{kY} = r^2\left[\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}\right]^{1/2}$$

5. $\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}\dfrac{1}{\Delta(\omega)}$ ; $\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$

6. For cantilever beam with load $P$ at the free end, $\delta_{max} = PL^3(3EI)^{-1}$

7. For cantilever beam with uniform load $w$, $\delta_{max} = wL^4(8EI)^{-1}$

**APPENDIX 2**/*LAMPIRAN 2*

## LAPLACE TRANSFORM PAIRS

| | $F(s)$ | $f(t)$ |
|---|---|---|
| 1 | $1$ | $\delta(t)$ = Dirac delta function |
| 2 | $\frac{1}{s}$ | $u(t)$ = unit step function |
| 3 | $\frac{1}{s^2}$ | $t$ |
| 4 | $\frac{1}{s^n}\ (n = 1, 2, \ldots)$ | $\frac{t^{n-1}}{(n-1)!}$ |
| 5 | $\frac{1}{s-a}$ | $e^{at}$ |
| 6 | $\frac{a}{s^2+a^2}$ | $\sin at$ |
| 7 | $\frac{s}{s^2+a^2}$ | $\cos at$ |
| 8 | $\frac{a}{s^2-a^2}$ | $\sinh at$ |
| 9 | $\frac{s}{s^2-a^2}$ | $\cosh at$ |
| 10 | $\frac{1}{(s^2+a^2)^2}$ | $\frac{1}{2a^3}(\sin at - at\cos at)$ |
| 11 | $\frac{s}{(s^2+a^2)^2}$ | $\frac{1}{2a}(t\sin at)$ |
| 12 | $\frac{a}{(s-b)^2+a^2}$ | $e^{bt}\sin at$ |
| 13 | $\frac{s-b}{(s-b)^2+a^2}$ | $e^{bt}\cos at$ |
| 14 | $\frac{1}{s^2+2\zeta\omega s+\omega^2}$ | $\frac{1}{\omega_d}e^{-\zeta\omega t}\sin\omega_d t,$ |
| 15 | $\frac{s+2\zeta\omega}{s^2+2\zeta\omega s+\omega^2}$ | $e^{-\zeta\omega t}\left[\cos\omega_d t + \frac{\zeta}{(1-\zeta^2)^{1/2}}\sin\omega_d t\right]$ where $\omega_d = \omega(1-\zeta^2)^{1/2}$ |

**APPENDIX 3**/*LAMPIRAN 3*

## Vibration-related Formulas

1. $\zeta = \dfrac{c}{2\omega_n m}$

2. $x(t) = e^{-\zeta\omega_n t}\left(A_1\cos(\omega_d t) + A_2\sin(\omega_d t)\right)$

3. $\omega_d = \sqrt{1-\zeta^2}\,\omega_n$

4. $x_p = X\sin(\omega t - \varphi)$, $X = \dfrac{F_0/k}{\left((1-r^2)^2 + (2\zeta r)^2\right)^{1/2}}$, $\varphi = \tan^{-1}\dfrac{2\zeta r}{1-r^2}$

5. $\dfrac{X}{Y} = \left[\dfrac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}\right]^{1/2}$ $\qquad \phi = \tan^{-1}\left[\dfrac{2\zeta r^3}{1+(4\zeta^2-1)r^2}\right]$

6. $\dfrac{F_T}{kY} = r^2\left[\dfrac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}\right]^{1/2}$

7. $TR = \left[\dfrac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}\right]^{1/2}$

8. $\dfrac{mX}{m_0 e} = \dfrac{r^2}{\left[(1-r^2)^2+(2\zeta r)^2\right]^{1/2}}$

9. $A^{-1} = \dfrac{1}{det(A)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

10. $\det(A) = ad - bc$

11. $f_d = \dfrac{1}{T}$

12. $\omega_d = \dfrac{2\pi n}{\Delta T}$

13. $\delta = \dfrac{1}{n}\ln\left(\dfrac{y_0}{y_n}\right)$

14. $\xi = \dfrac{1}{\sqrt{1+\left(\dfrac{2\pi}{\delta}\right)^2}}$

**-000000000-**